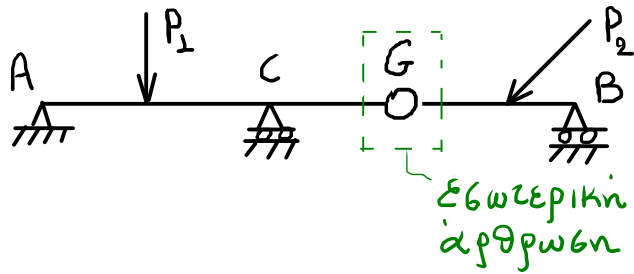


Μηχανική Ι

Δοκός τύπου Gerber:



$$\left. \begin{array}{l} (+) \rightarrow \sum F_x = 0 \\ (+) \uparrow \sum F_y = 0 \\ \curvearrowright \sum M_A = 0 \end{array} \right\} \begin{array}{l} 3 \text{ εξ. ισογ.} < 4 : \text{αριθμός αντιστάσεων} \end{array}$$

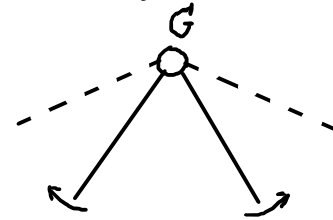
Υπερστατικό πρόβλημα
βαθμός υπερστατικότητας $4 - 3 = 1$

Επιπλέον εξίσωση

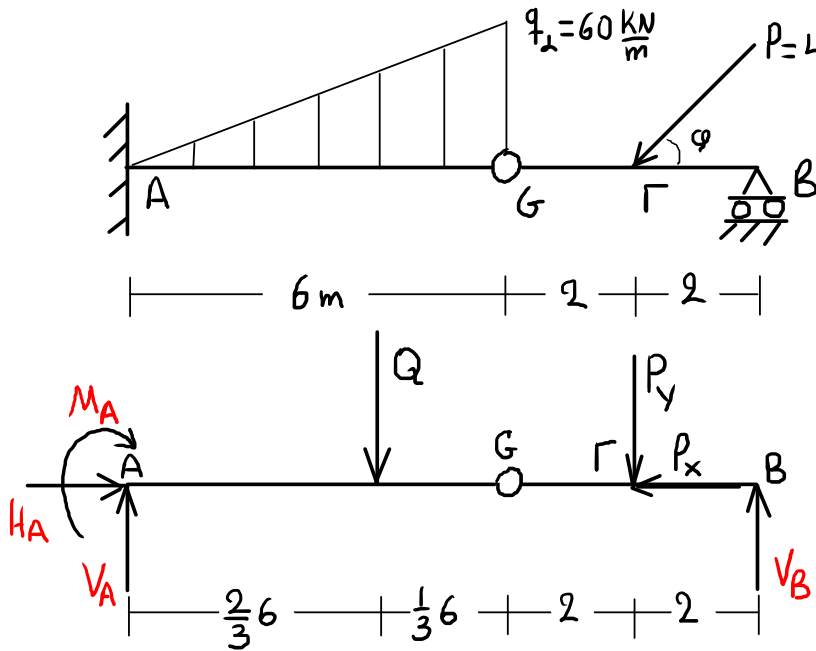
$\sum M_G^{\Delta I A} = 0$

 : βασική συνθήκη ισορροπίας
δοκού Gerber στο επίπεδο

Επεξήγηση εξωτερικής άρθρωσης:
παράδειγμα διαβήτη



Παράδειγμα:



$$P = 40 \text{ kN} \quad \eta \gamma \varphi = 0,6, \quad \varsigma \nu \varphi = 0,8$$

$$Q = \frac{1}{2} \cdot 6 q_x = \frac{1}{2} \cdot 6 \cdot 60 = 180 \text{ kN}$$

$$P_x = P \varsigma \nu \varphi = 40 \cdot 0,8 = 32 \text{ kN}$$

$$P_y = P \eta \gamma \varphi = 40 \cdot 0,6 = 24 \text{ kN}$$

$$(+)\Rightarrow \sum F_x = 0 \Rightarrow H_A - P_x = 0 \Rightarrow \underline{H_A = 32 \text{ kN}} \quad (1)$$

$$+\uparrow \sum F_y = 0 \Rightarrow V_A - Q - P_y + V_B = 0 \Rightarrow \underline{V_A + V_B = 204} \quad (2)$$

$$+\circlearrowleft \sum M_A = 0 \Rightarrow -M_A - Q \cdot 4 - P_y \cdot 8 + V_B \cdot 10 = 0 \Rightarrow \underline{M_A - 10V_B = -912} \quad (3)$$

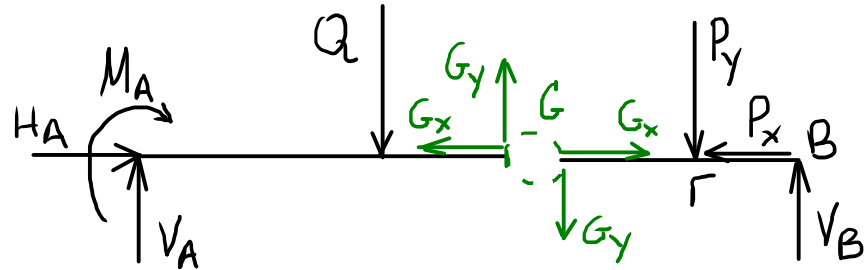
$$+\circlearrowleft \sum M_G^{AG/GB} = 0 : \text{GB} \quad +\circlearrowleft \sum M_G^{GB} = 0 \Rightarrow -P_y \cdot 2 + V_B \cdot 4 = 0 \Rightarrow V_B = \frac{2P_y}{4} = 12 \text{ kN} \quad (4)$$

$$(2) \stackrel{(4)}{\Rightarrow} V_A = 204 - 12 = 192 \text{ kN}, \quad (3) \stackrel{(4)}{\Rightarrow} M_A = -912 + 120 = -792 \text{ kNm}$$

$$+\circlearrowleft \sum M_G^{AG} = 0 \Rightarrow -V_A \cdot 6 - M_A + Q \cdot 2 = 0 \Rightarrow -192 \cdot 6 + 792 + 180 \cdot 2 = 0 \Rightarrow 0 = 0 \quad \text{OK!}$$

Εύρεση εσωτερικά μεταβιβασόμενων αντιστάσεων:

Δ.Ε.Σ αφαιρούμε την εσωτερική άρθρωση και βάζουμε την θέση της συμπεριωνούμε τις εσωτερικές αντιστάσεις.



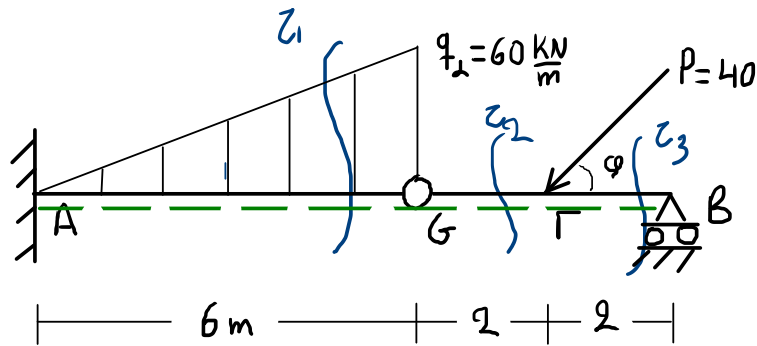
$$AG: \overset{(+)}{\rightarrow} \sum F_x = 0 \Rightarrow H_A - G_x = 0 \Rightarrow G_x = H_A \Rightarrow G_x = 32 \text{ kN}$$

$$(+)\uparrow \sum F_y = 0 \Rightarrow V_A - Q + G_y = 0 \Rightarrow G_y = Q - V_A = 180 - 192 \Rightarrow G_y = -12 \text{ kN}$$

$$GB: \overset{(+)}{\rightarrow} \sum F_x = 0 \Rightarrow G_x - P_x = 0 \Rightarrow 32 - 32 = 0 \Rightarrow 0 = 0 \text{ } \checkmark$$

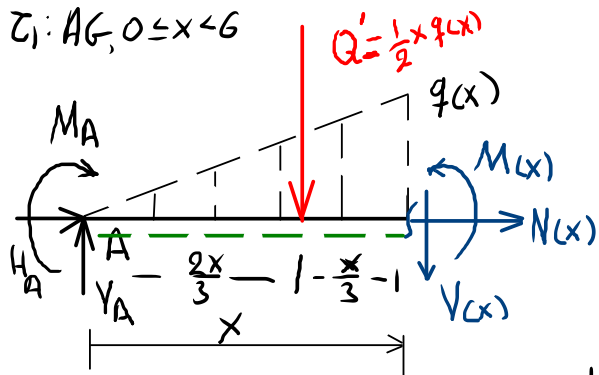
$$(+)\uparrow \sum F_y = 0 \Rightarrow -G_y - P_y + V_B = 0 \Rightarrow 12 - 24 + 12 = 0 \Rightarrow 0 = 0 \text{ } \checkmark$$

Διαγράμματα ενταξικών μεγεθών [N], [V], [M]



2ος τρόπος: Αντιμετώπιση ως ενιαία δοκό

Σ1: AG, $0 \leq x < 6$



$$q(x) = \frac{q_1}{6} \Rightarrow q(x) = \frac{60}{6}x \Rightarrow q(x) = 10x$$

$$Q' = \frac{1}{2}x(10x) = 5x^2$$

$$\uparrow \sum F_x = 0 \Rightarrow H_A + N(x) = 0 \Rightarrow N(x) = -H_A = -32 \text{ kN}$$

$$\uparrow \sum F_y = 0 \Rightarrow V_A - Q' - V(x) = 0 \Rightarrow V(x) = 192 - 5x^2 \quad \left| \begin{array}{l} A: x=0 \\ G: x=6 \end{array} \right.$$

$$A: V(0) = 192 \text{ kN}$$

$$G: V(6) = 12 \text{ kN}$$

$$\frac{dV(x)}{dx} = 0 \Rightarrow -10x = 0 \Rightarrow x = 0 \text{ από } x \geq 0$$

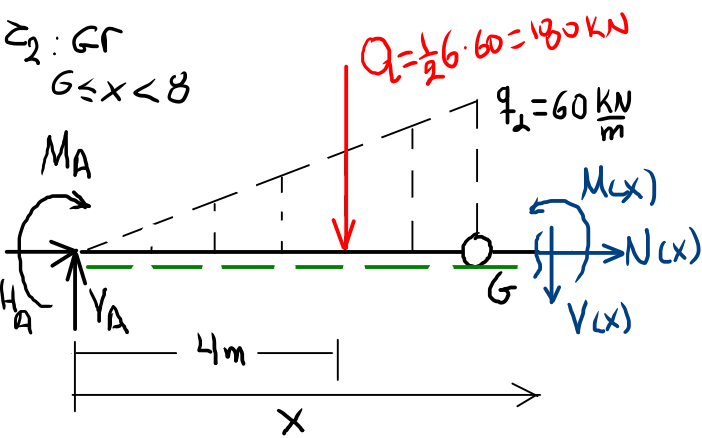
$$\frac{d^2V(x)}{dx^2} = -10 < 0 \text{ κοίτη και } x=0$$

$$\uparrow \sum M_x = 0 \Rightarrow M(x) + Q' \cdot \frac{x}{3} - V_A \cdot x - M_A = 0$$

$$\Rightarrow M(x) = -792 + 192x - \frac{5}{3}x^3 \quad \left| \begin{array}{l} A: x=0, M(0) = -792 \text{ kNm} \\ G: x=6, M(6) = -792 + 1152 - 360 = 0 \end{array} \right.$$

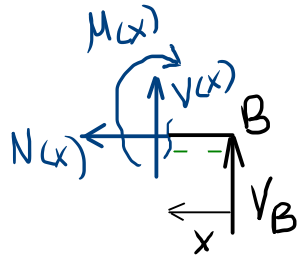
$$\frac{dM(x)}{dx} = 0 \Rightarrow 192 - 5x^2 = 0 \Rightarrow x = \pm \sqrt{\frac{192}{5}} = \pm 6.2$$

$$\frac{d^2M(x)}{dx^2} = -10x < 0$$



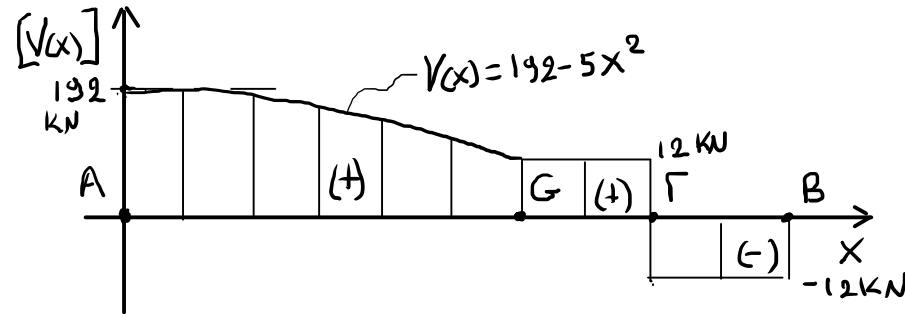
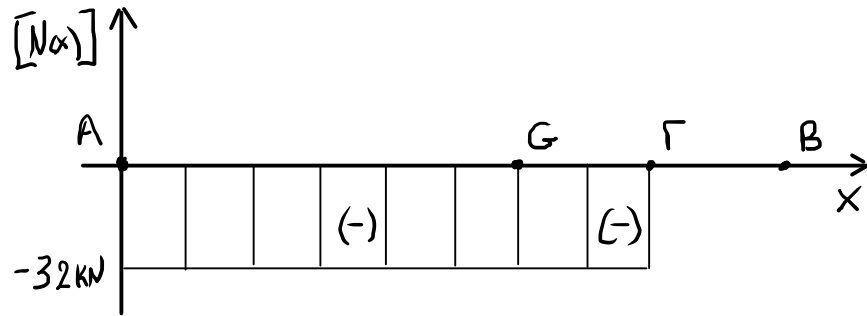
$$\begin{aligned}
 (+) \sum F_x = 0 &\Rightarrow H_A + N(x) = 0 \Rightarrow N(x) = -32 \text{ kN} \\
 (+) \uparrow \sum F_y = 0 &\Rightarrow V_A - Q - V(x) = 0 \Rightarrow \\
 &V(x) = 192 - 180 = 12 \text{ kN} \\
 (+) \sum M_x = 0 &\Rightarrow -M_A - V_A \cdot x + Q(x-4) + M(x) = 0 \\
 \Rightarrow M(x) &= -72 + 12x \quad \left| \begin{array}{l} G: x=6 \Rightarrow M(6) = 0 \\ F: x=8 \Rightarrow M(8) = 24 \text{ kNm} \end{array} \right.
 \end{aligned}$$

$\Sigma_3: FB, 0 \leq x < 2$

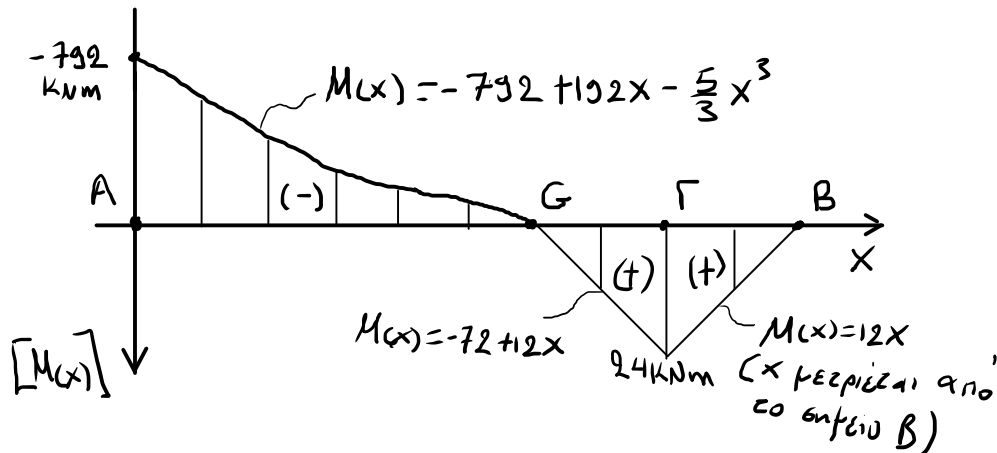


$$\begin{aligned}
 (+) \sum F_x = 0 &\Rightarrow -N(x) = 0 \Rightarrow N(x) = 0 \\
 (+) \uparrow \sum F_y = 0 &\Rightarrow V(x) + V_B = 0 \Rightarrow V(x) = -12 \text{ kN} \\
 (+) \sum M_x = 0 &\Rightarrow -M(x) + V_B \cdot x = 0 \\
 \Rightarrow M(x) &= 12 \cdot x \quad \left| \begin{array}{l} B: x=0, M(0) = 0 \\ F: x=2, M(2) = 24 \text{ kNm} \end{array} \right.
 \end{aligned}$$

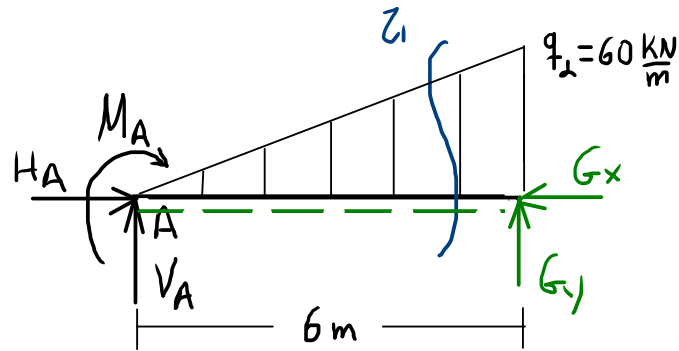
Διαγράμματα εσωτερικών μεγεθών $[N(x)]$, $[V(x)]$, $[M(x)]$



$$V(x) = \frac{dM(x)}{dx}$$

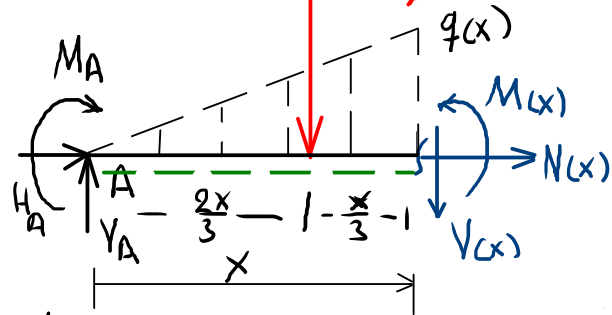


2ος τρόπος: αναμεμειγμένα ως δύο δοκοί, αφαιρώντας την εσωτερική άρθρωση στο σημείο Γ, παίρνουμε τη δοκό ΑΒ και ΓΒ όπου βγαίνει Ε.Σ. Γνωρίζουμε τις G_x, G_y



1: ΑΓ, $0 \leq x < 6$

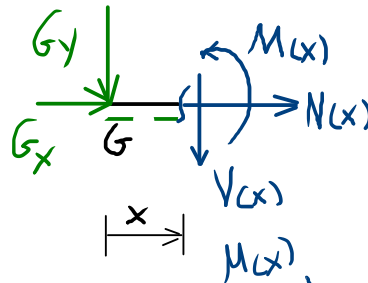
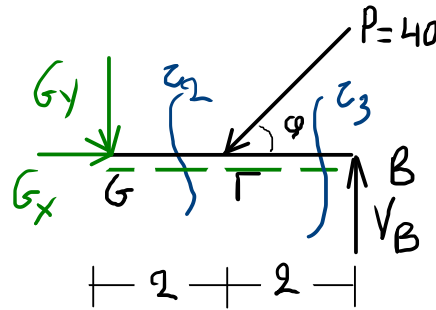
$$Q' = \frac{1}{2} \times 60 \times x$$



$$N(x) = -32 \text{ kN}$$

$$V(x) = 192 - 5x^2$$

$$M(x) = -792 + 192x - \frac{5}{3}x^3$$



$$N(x) = -G_x = -32 \text{ kN}$$

$$V(x) = -G_y = 12 \text{ kN}$$

$$M(x) = -G_y \cdot x = +12x$$

$$| \text{Γ: } x=0, M(0)=0$$

$$| \text{B: } x=4, M(4)=24 \text{ kNm}$$

$$N(x) = 0$$

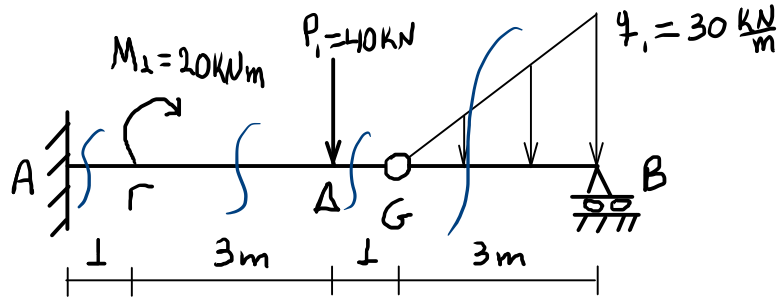
$$V(x) = -12$$

$$M(x) = 12x \quad | \text{Γ: } x=2, M(2)=24$$

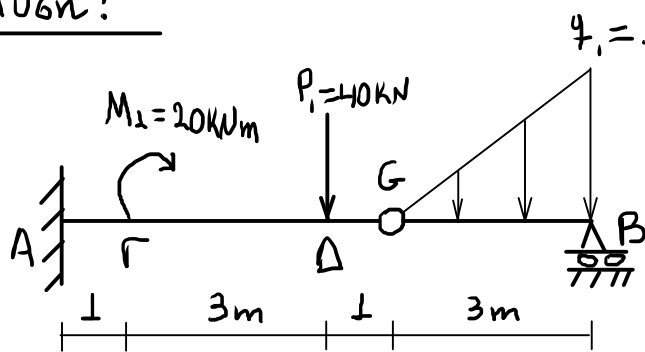
$$| \text{B: } x=0, M(0)=0$$

Άσκηση:

Να υπολογιστούν οι αντιδράσεις στις γωνίες και οι εσωτερικές αντιδράσεις του δοκού Gerber του σχήματος.



Λύση:



$$Q = \frac{1}{2} \cdot 3 \cdot 30 = 45 \text{ kN}$$

$$(+)\rightarrow \sum F_x = 0 \Rightarrow \underline{H_A = 0} \quad (1)$$

$$(+)\uparrow \sum F_y = 0 \Rightarrow V_A + V_B - P_1 - Q = 0 \Rightarrow \underline{V_A + V_B = 85} \quad (2)$$

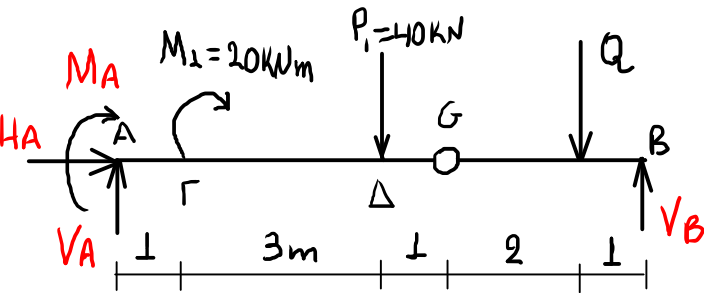
$$\begin{aligned} (+)\rightarrow \sum M_A = 0 &\Rightarrow -M_A - M_1 - P_1 \cdot 4 - Q \cdot 7 + V_B \cdot 8 = 0 \\ &\Rightarrow -M_A - 20 - 40 \cdot 4 - 45 \cdot 7 + V_B \cdot 8 = 0 \end{aligned}$$

$$\Rightarrow \underline{M_A - 8V_B = -495} \quad (3)$$

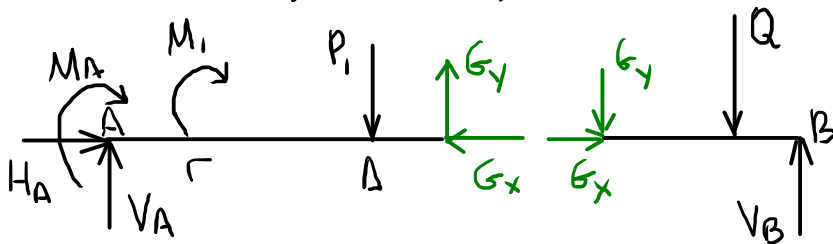
$$(+)\rightarrow \sum M_G = 0 \Rightarrow -Q \cdot 2 + V_B \cdot 3 = 0 \Rightarrow V_B = \frac{2}{3} \cdot 45 = 30 \text{ kN} \quad (4)$$

$$(3) \xrightarrow{(4)} M_A = -495 + 8 \cdot 30 = -255 \text{ kNm}$$

$$(2) \xrightarrow{(4)} V_A = 85 - 30 = 55 \text{ kN}$$



Εύρεση εσωτερικών αντιδράσεων:



Τμήμα GB:

$$(+)\rightarrow \sum F_x = 0 \Rightarrow \underline{G_x = 0}$$

$$\begin{aligned} (+)\uparrow \sum F_y = 0 &\Rightarrow -G_y - Q + V_B = 0 \\ &\Rightarrow G_y = V_B - Q = 30 - 45 \\ &\Rightarrow \underline{G_y = -15 \text{ kN}} \end{aligned}$$

Τμήμα AG:

$$\begin{aligned} &(+)\rightarrow \sum F_x = 0 \\ &(+)\uparrow \sum F_y = 0 \end{aligned} \left\{ \begin{array}{l} \text{θα πρέπει να ικανοποιούνται} \end{array} \right.$$