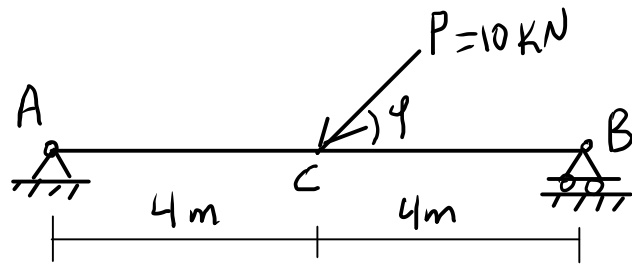


Εντασιακά μεγέθη $[N]$, $[V]$, $[M]$

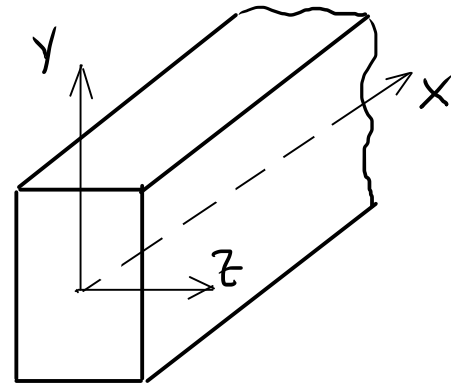


$$\eta \phi = 0,6$$

$$\cos \phi = 0,8$$

$$P_x = P \cos \phi = 8 \text{ kN}$$

$$P_y = P \eta \phi = 6 \text{ kN}$$



$[N]$: αξονικές δυνάμεις

$[V]$: κάθετες δυνάμεις

$[M]$: καμπτικές ροπές

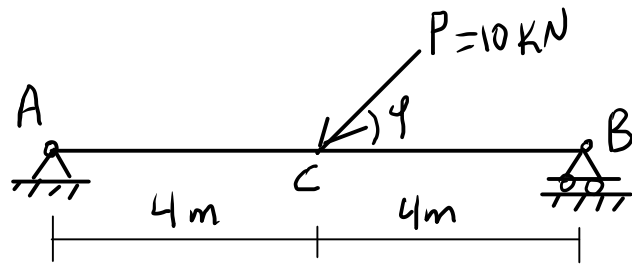
$$(\rightarrow) \sum F_x = 0 \Rightarrow H_A - P_x = 0 \Rightarrow \underline{H_A = P_x = 8 \text{ kN}} \quad (1)$$

$$(\uparrow) \sum F_y = 0 \Rightarrow V_A - P_y + V_B = 0 \Rightarrow V_A + V_B = P_y = 6 \text{ kN} \quad (2)$$

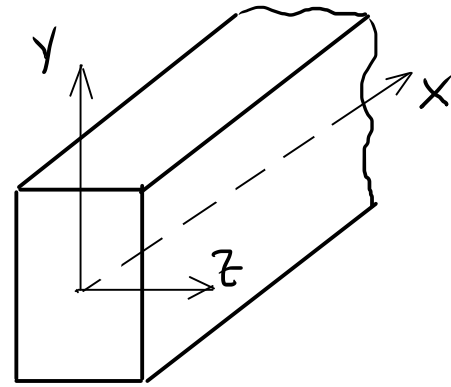
$$(\curvearrowright) \sum M_A = 0 \Rightarrow -P_y \cdot 4 + V_B \cdot 8 = 0 \Rightarrow V_B = P_y \frac{4}{8} = \frac{P_y}{2} \Rightarrow \underline{V_B = 3 \text{ kN}} \quad (3)$$

$$(2) \xrightarrow{(3)} V_A + 3 = 6 \Rightarrow \underline{V_A = 3 \text{ kN}}$$

Εντασιακά μεγέθη $[N]$, $[V]$, $[M]$



$$\begin{aligned} n\phi\phi &= 0,6 \\ \cos\phi &= 0,8 \\ P_x &= P \cos\phi = 8 \text{ kN} \\ P_y &= P \cdot n\phi\phi = 6 \text{ kN} \end{aligned}$$



$[N]$: αξονικές δυνάμεις

$[V]$: ροπές δυνάμεις

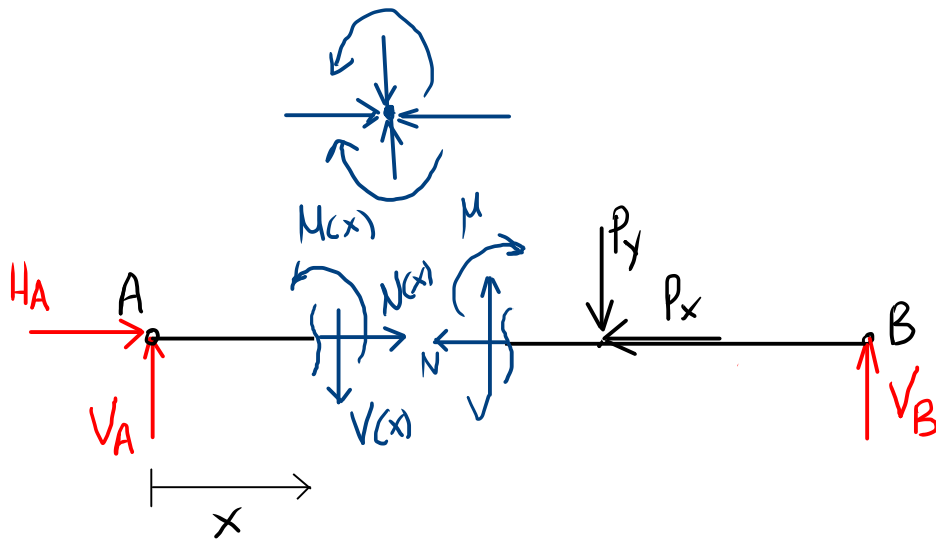
$[M]$: καμπτικές ροπές

$$\rightarrow \sum F_x = 0 \Rightarrow H_A - P_x = 0 \Rightarrow \underline{H_A = P_x = 8 \text{ kN}} \quad (1)$$

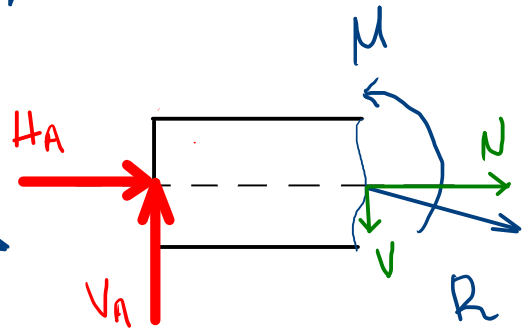
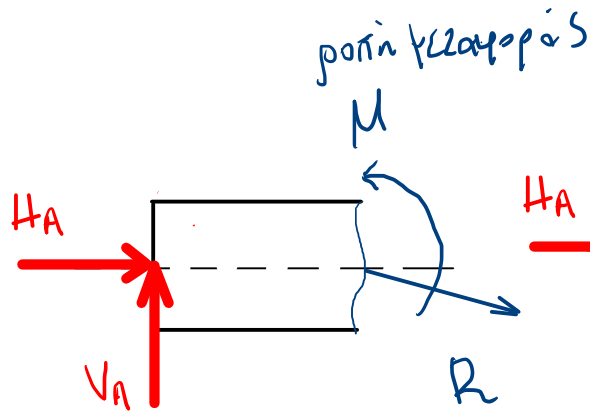
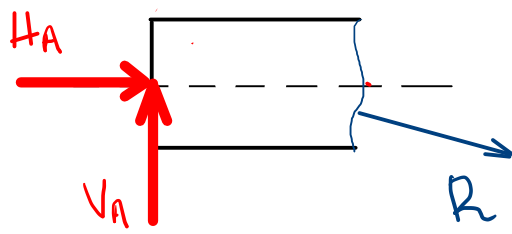
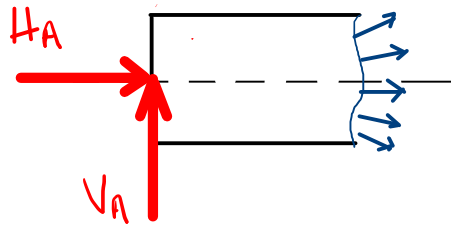
$$\uparrow \sum F_y = 0 \Rightarrow V_A - P_y + V_B = 0 \Rightarrow V_A + V_B = P_y = 6 \text{ kN} \quad (2)$$

$$\curvearrowright \sum M_A = 0 \Rightarrow -P_y \cdot 4 + V_B \cdot 8 = 0 \Rightarrow V_B = P_y \frac{4}{8} = \frac{P_y}{2} \Rightarrow \underline{V_B = 3 \text{ kN}} \quad (3)$$

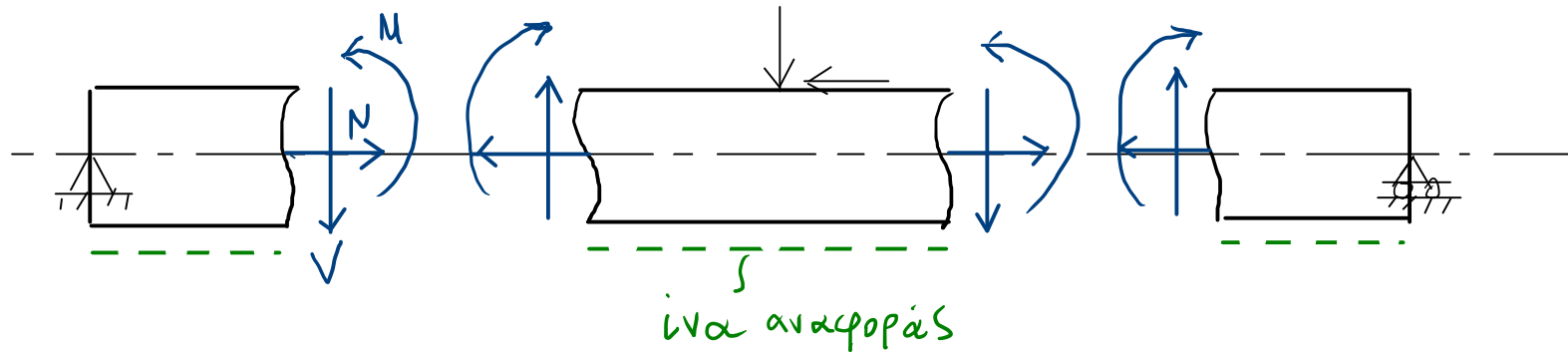
$$(2) \xrightarrow{(3)} V_A + 3 = 6 \Rightarrow \underline{V_A = 3 \text{ kN}}$$



* Τα δύο αποκτοζόμενα κομμάτια θα πρέπει να ισορροπούν με την ανάπτυξη των εσωτερικών εντατικών μεγεθών με διαφορά στο επίπεδο της ροπής



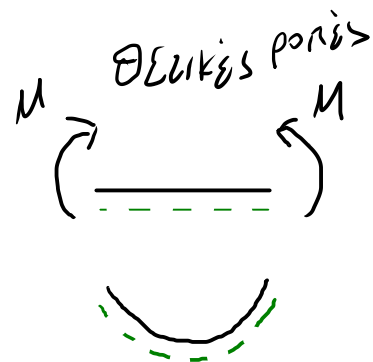
Σύμβολα θετικών ενταζτικών μεγεθών

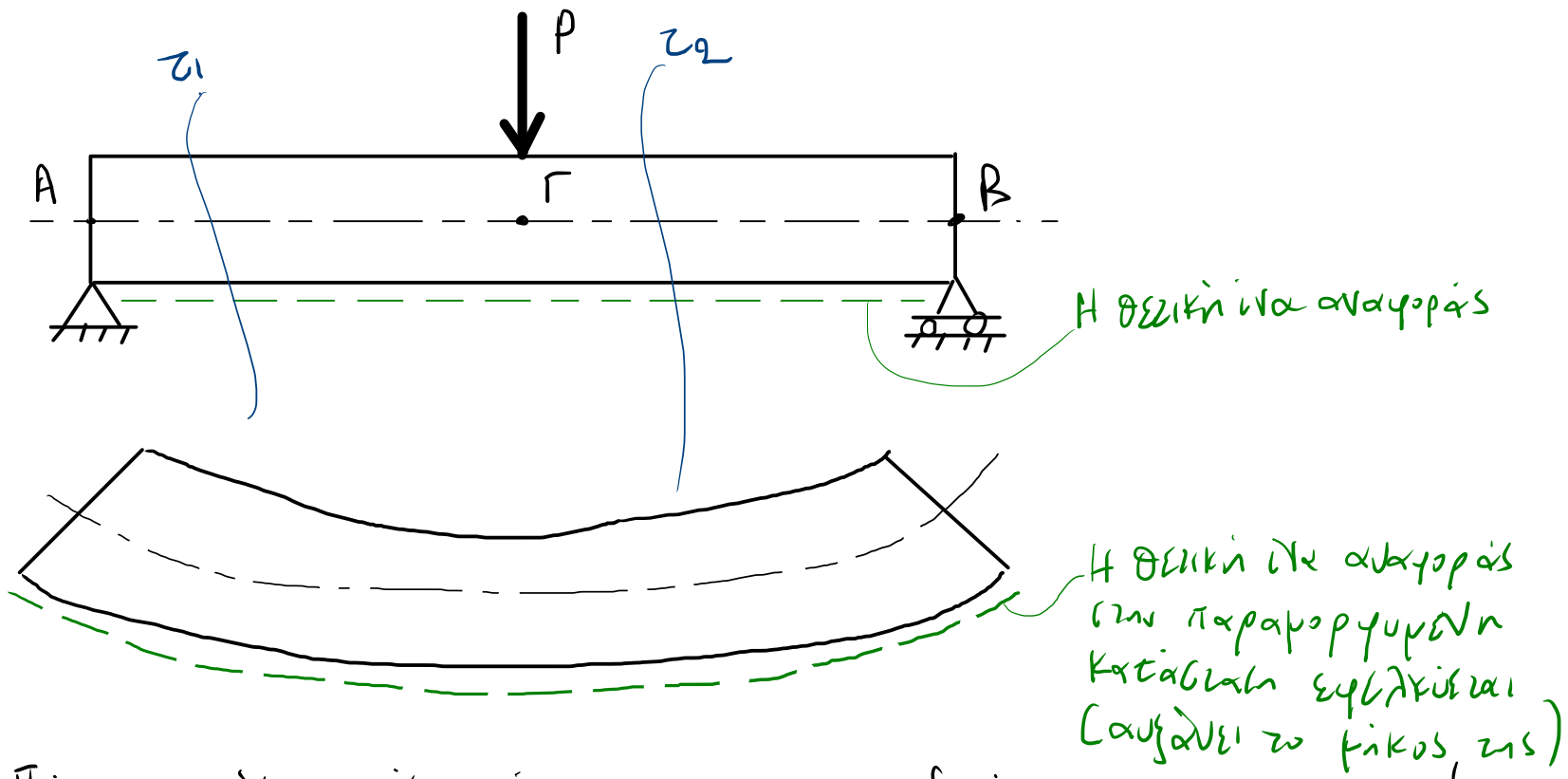


$[N(x)]$: δείχνει να εφελκύζει το αποκοπόμενο κομμάτι

$[V(x)]$: δείχνει να το περιβρέγει ωρολογιακά

$[M(x)]$: δείχνει να εφελκύβει την δα αναφοράς



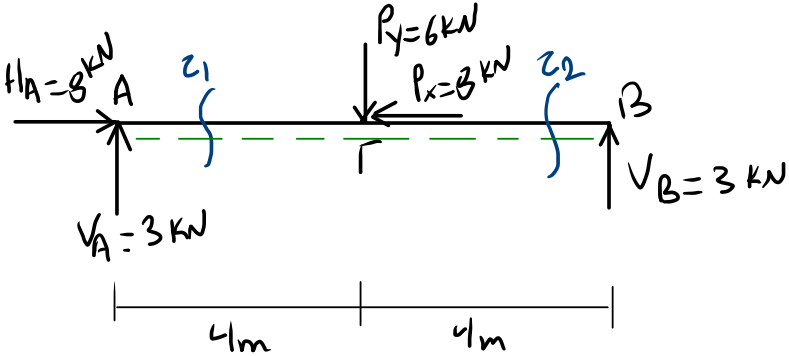


* Πόσες νοητές τομές πρέπει να πάρουμε στη δοκό για τον υπολογισμό των εντατικών μεγεθών;

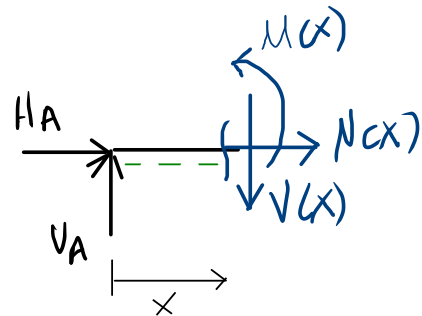
Απάντηση: Παιρνουμε τόσες τομές όσες το πλήθος των οριζόντιων αβιχέλιας φορτίου
μειών για (και για λιγότερα από το πλήθος των οριζόντιων αβιχέλιας φορτίου)

Σημεία αβιχέλιας φορτίου: είναι τα οριζόντια της δοκού όπου ασκούνται

συγκεντρωμένες δυνάμεις ή ροπές. Λίγο πριν και λίγο μετά το σημείο αβιχέλιας φορτίου
αλλάζουν τα εντατικά μεγέθη



Topin z_1 : $0 < x < 4$ (AΓ)



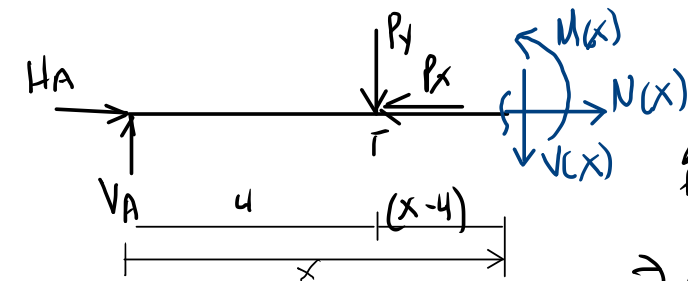
$$(+)\rightarrow \sum F_x = 0 \Rightarrow H_A + N(x) = 0 \Rightarrow N(x) = -H_A \Rightarrow \underline{N(x) = -8 \text{ kN}}$$

$$(+)\uparrow \sum F_y = 0 \Rightarrow V_A - V(x) = 0 \Rightarrow V(x) = V_A \Rightarrow \underline{V(x) = 3 \text{ kN}}$$

$$\curvearrowright \sum M_x = 0 \Rightarrow -V_A \cdot x + M(x) = 0 \Rightarrow M(x) = V_A \cdot x$$

$$\Rightarrow M(x) = 3 \cdot x \quad \left| \begin{array}{l} A^+: x = 0^+ \Rightarrow M(0^+) = 0 \text{ kN}\cdot\text{m} \\ \Gamma^-: x \rightarrow 4 \Rightarrow M(4^-) = 3 \cdot 4 = 12 \text{ kN}\cdot\text{m} \end{array} \right.$$

Topin z_2 : $4 \leq x < 8$ (ΓB)



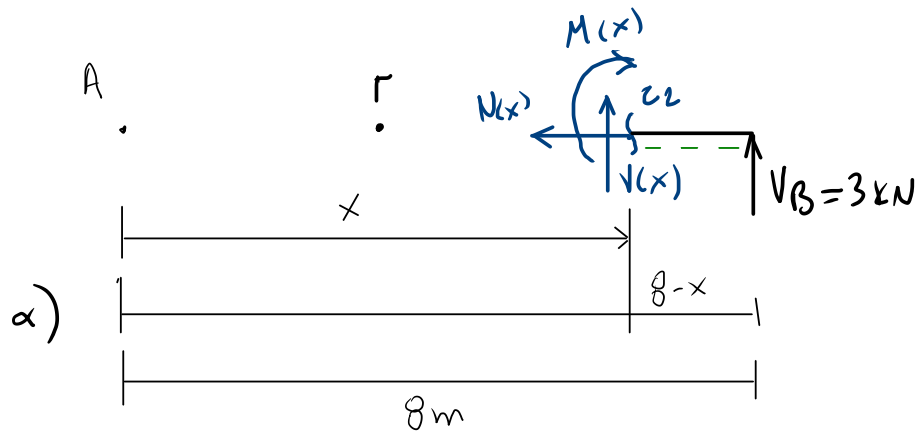
$$(+)\rightarrow \sum F_x = 0 \Rightarrow H_A - P_x + N(x) = 0 \Rightarrow \underline{N(x) = P_x - H_A = 3 - 8 = -5 \text{ kN}}$$

$$(+)\uparrow \sum F_y = 0 \Rightarrow V_A - P_y - V(x) = 0 \Rightarrow \underline{V(x) = V_A - P_y = 3 - 6 = -3 \text{ kN}}$$

$$\curvearrowright \sum M_x = 0 \Rightarrow -V_A \cdot x + P_y \cdot (x-4) + M(x) = 0 \Rightarrow$$

$$\Rightarrow M(x) = 3 \cdot x - 6 \cdot (x-4) \Rightarrow$$

$$\Rightarrow \underline{M(x) = -3x + 24} \quad \left| \begin{array}{l} \Gamma: x = 4, M(4) = 12 \text{ kN} \\ B: x = 8, M(8) = 0 \text{ kN} \end{array} \right.$$



Top in z_2 (RB)



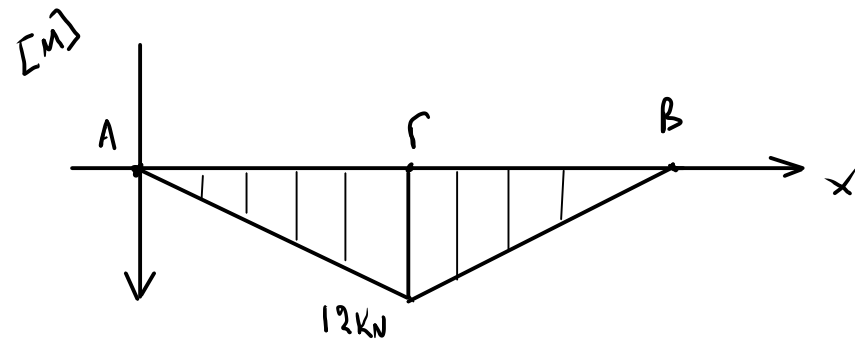
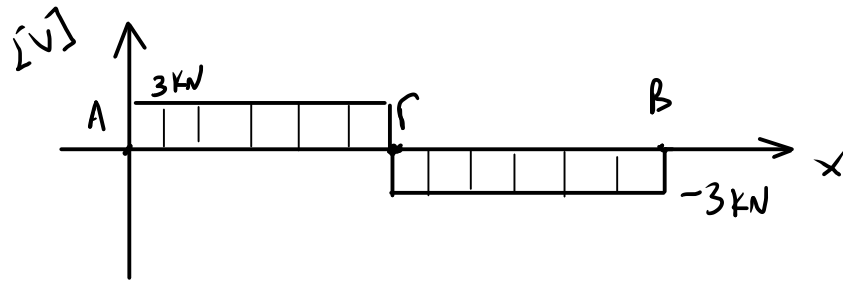
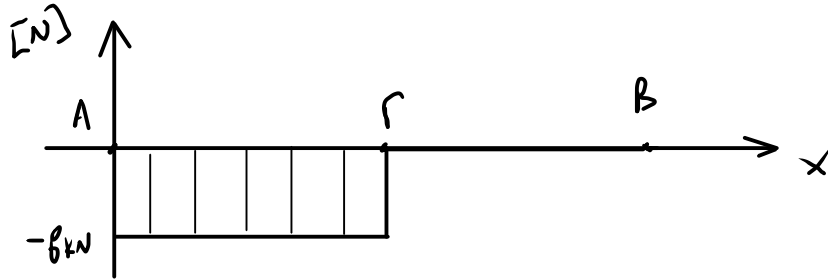
α) $4 \leq x < 8$

$$\begin{aligned} \textcircled{+} \sum F_x &= 0 \Rightarrow -N(x) = 0 \Rightarrow N(x) = 0 \\ \textcircled{+} \sum F_y &= 0 \Rightarrow V(x) + V_B = 0 \Rightarrow V(x) = -V_B \Rightarrow V(x) = -3 \text{ kN} \\ \textcircled{+} \sum M_x &= 0 \Rightarrow V_B \cdot (8-x) - M(x) = 0 \Rightarrow M(x) = 3(8-x) \\ &\Rightarrow M(x) = 24 - 3x \end{aligned}$$

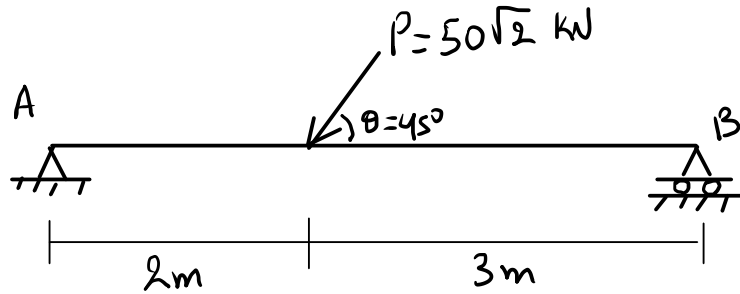
β) $0 < x \leq 4$

$$\begin{aligned} \textcircled{+} \sum F_x &= 0, \textcircled{+} \sum F_y = 0 \quad \text{ο ίδιος προηγούμενος} \\ \textcircled{+} \sum M_x &= 0 \Rightarrow V_B \cdot x - M(x) = 0 \Rightarrow M(x) = 3 \cdot x \end{aligned} \quad \left| \begin{array}{l} \text{B: } x=0, M(0)=0 \\ \text{Γ: } x=4, M(4)=12 \text{ kN} \end{array} \right.$$

Διαγράμματα $[N]$, $[V]$, $[M]$

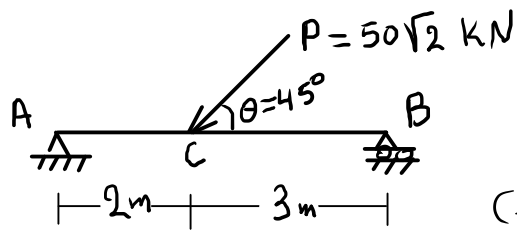


Άσκηση:



Για την αμφώριστη δοκό του σχήματος
ζητούνται: α) να υπολογιστούν οι
δυνάμεις αντίδρασεις και β)
να σχεδιαστείτε τα διαγράμματα
των ενταξικών μεγεθών. $[N]$, $[V]$, $[M]$

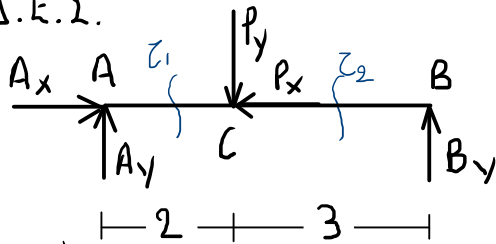
Αντικείμενο δοκός με ενδιαμέσο συγκεντρωμένο φορτίο:



$$P_x = P \cdot \cos \theta = 50\sqrt{2} \cdot \frac{\sqrt{2}}{2} = 50 \text{ kN}$$

$$P_y = P \cdot \sin \theta = 50\sqrt{2} \cdot \frac{\sqrt{2}}{2} = 50 \text{ kN}$$

Δ.Ε.Σ.



$$(+)\sum F_x = 0 \Rightarrow A_x - P_x = 0 \Rightarrow A_x = P_x \Rightarrow A_x = 50 \text{ kN} \quad (1)$$

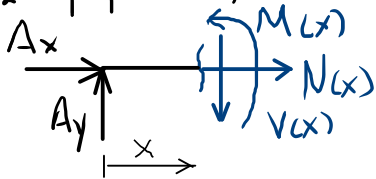
$$(+)\uparrow \sum F_y = 0 \Rightarrow A_y - P_y + B_y = 0 \Rightarrow A_y + B_y = 50 \quad (2)$$

$$\curvearrowright \sum M_A = 0 \Rightarrow -P_y \cdot 2 + B_y \cdot 5 = 0 \Rightarrow B_y = \frac{50 \cdot 2}{5} = 20 \text{ kN} \quad (3)$$

$$(2) \xrightarrow{(3)} A_y = 50 - 20 = 30 \text{ kN}$$

Τόμεις:

Σ_1 : τμήμα AC, $0 \leq x < 2$



$$(+)\sum F_x = 0 \Rightarrow A_x + N(x) = 0 \Rightarrow N(x) = -50 \text{ kN}$$

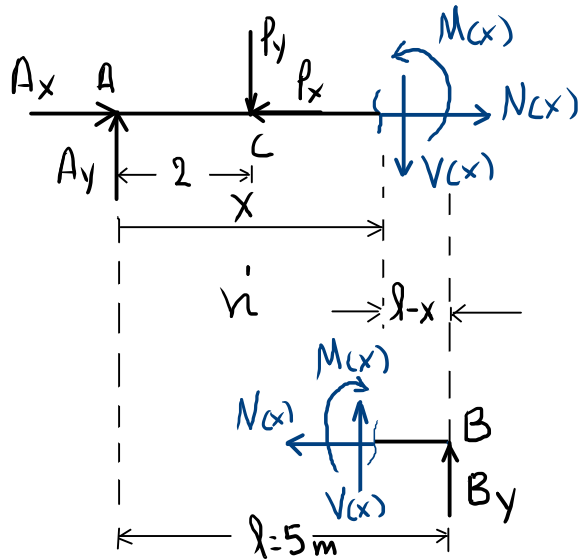
$$(+)\uparrow \sum F_y = 0 \Rightarrow A_y - V(x) = 0 \Rightarrow V(x) = 30 \text{ kN}$$

$$\curvearrowright \sum M_x = 0 \Rightarrow -A_y \cdot x + M(x) = 0 \Rightarrow M(x) = 30x$$

$$A: x=0 \Rightarrow M(0) = 0$$

$$C^-: x \rightarrow 2 \Rightarrow M(2) = 60 \text{ kN.m}$$

Z_2 : $Z \text{ από } CB, 2 \leq x \leq 5$



$$\begin{aligned} (+) \rightarrow \sum F_x = 0 &\Rightarrow A_x - P_x + N(x) = 0 \Rightarrow N(x) = P_x - A_x \\ &\Rightarrow \underline{N(x) = 50 - 50 = 0} \end{aligned}$$

$$\begin{aligned} (+) \uparrow \sum F_y = 0 &\Rightarrow A_y - P_y - V(x) = 0 \Rightarrow V(x) = A_y - P_y \\ &\Rightarrow \underline{V(x) = 30 - 50 = -20 \text{ kN}} \end{aligned}$$

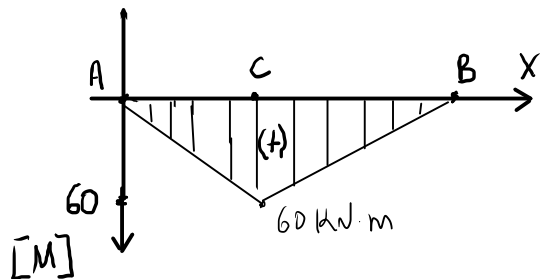
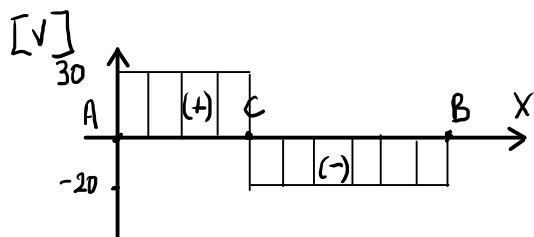
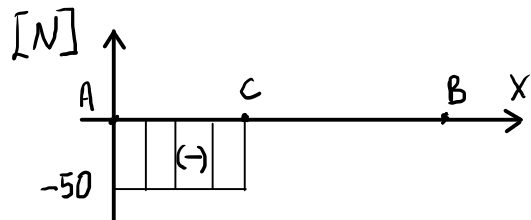
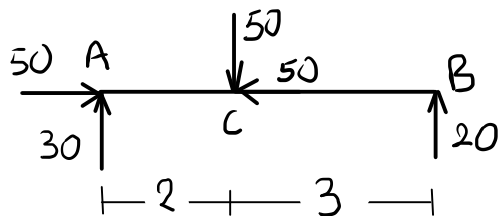
$$\begin{aligned} \curvearrowright \sum M_x = 0 &\Rightarrow -A_y \cdot x + P_y \cdot (x-2) + M(x) = 0 \\ &\Rightarrow M(x) = x(A_y - P_y) + 2P_y \\ &\Rightarrow M(x) = x(30 - 50) + 2 \cdot 50 \end{aligned}$$

$$\Rightarrow \underline{M(x) = -20 \cdot x + 100} \quad \left| \begin{array}{l} C: x=2 \Rightarrow M(2) = 60 \text{ kN}\cdot\text{m} \\ B: x=5 \Rightarrow M(5) = 0 \text{ kN}\cdot\text{m} \end{array} \right.$$

ν Δ.Ε.Σ. δεξι κομμάτι

$$\begin{aligned} (+) \sum M_x = 0 &\Rightarrow M(x) - B_y \cdot (5-x) = 0 \Rightarrow M(x) = -x \cdot B_y + 5B_y \Rightarrow \\ &\Rightarrow M(x) = -20x + 100 \end{aligned}$$

Σχεδίαση διαγράμμάτων $[N]$, $[V]$, $[M]$

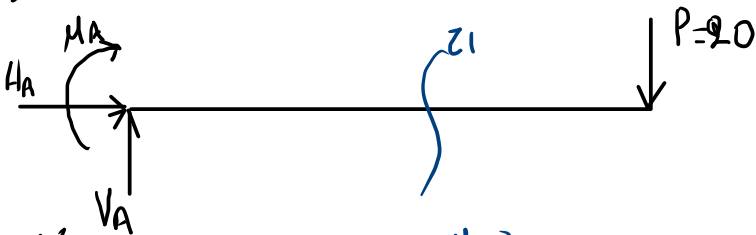


Παράδειγμα:

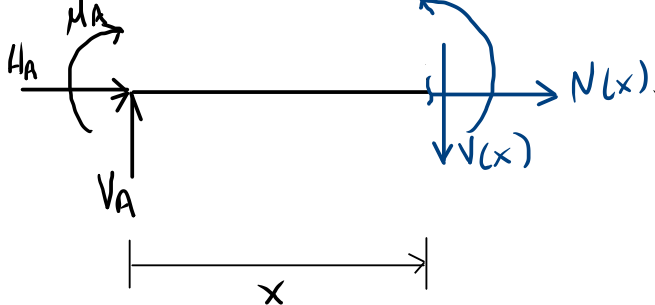
πρόβλημα



Δε.Σ.



Δε.Σ.



$$(+)\rightarrow \sum F_x = 0 \Rightarrow \underline{H_A = 0}$$

$$(+)\uparrow \sum F_y = 0 \Rightarrow V_A - P = 0 \Rightarrow V_A = P \Rightarrow \underline{V_A = 20}$$

$$\curvearrowright \sum M_A = 0 \Rightarrow -20 \cdot 4 - M_A = 0 \Rightarrow \underline{M_A = -80 \text{ kN}\cdot\text{m}}$$

Τομή z_1 : $0 < x < l$ (AB):

$$(+)\rightarrow \sum F_x = 0 \Rightarrow H_A + N(x) = 0 \Rightarrow N(x) = -H_A \Rightarrow \underline{N(x) = 0}$$

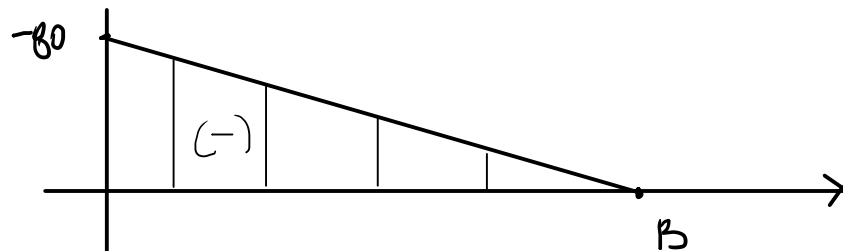
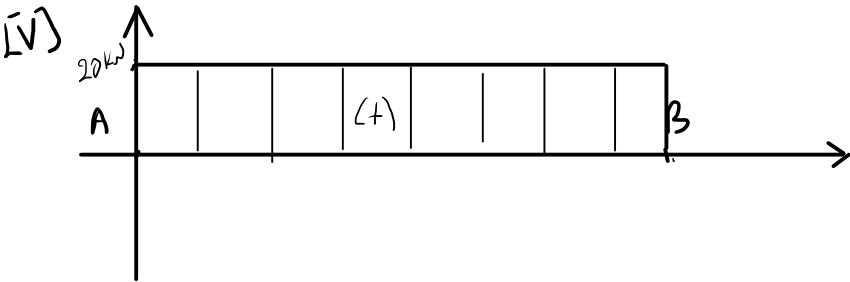
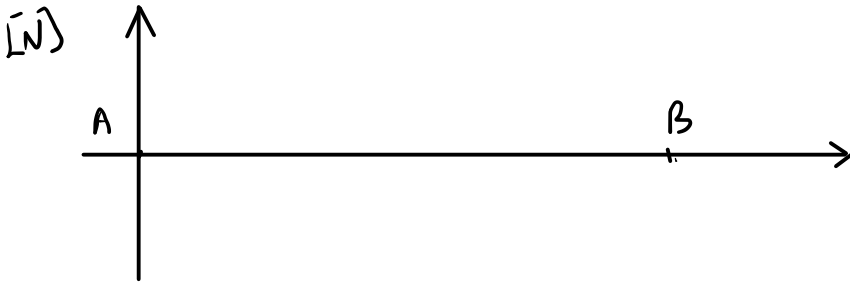
$$(+)\uparrow \sum F_y = 0 \Rightarrow V_A - V(x) = 0 \Rightarrow V(x) = V_A \Rightarrow \underline{V(x) = 20 \text{ kN}}$$

$$\curvearrowright \sum M_x = 0 \Rightarrow -V_A \cdot x - M_A + M(x) = 0$$

$$\Rightarrow M(x) = V_A \cdot x + M_A$$

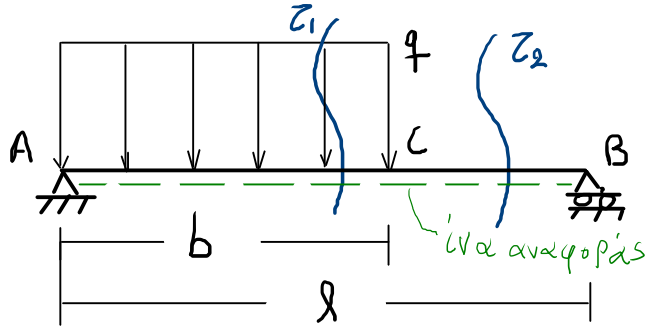
$$\Rightarrow M(x) = 20 \cdot x - 80 \quad \left\{ \begin{array}{l} A^+ : x=0, M(0) = -80 \\ B^- : x=4, M(4) = 80 - 80 = 0 \end{array} \right.$$

Διαγράμματα $[N]$, $[V]$, $[M]$



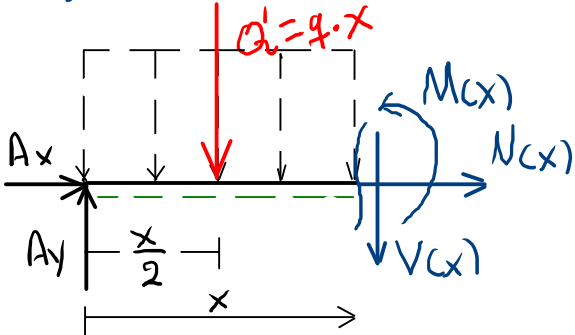
Η μέγιστη (κατά μέτρο) ροπή
εμφανίζεται στο σημείο A^+
(ανω πάκλωση) $|M_{\max}| = 80 \text{ kN}\cdot\text{m}$

Ανάλυση των πιγν καταμετρημένων φορτίων κατά τη σχεδίαση των δια-
 γραμμάτων $[N]$, $[V]$, $[M]$:

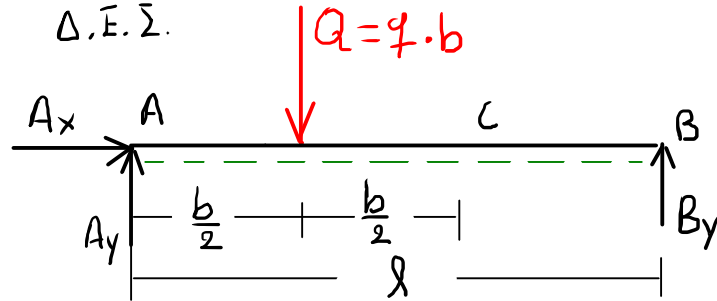
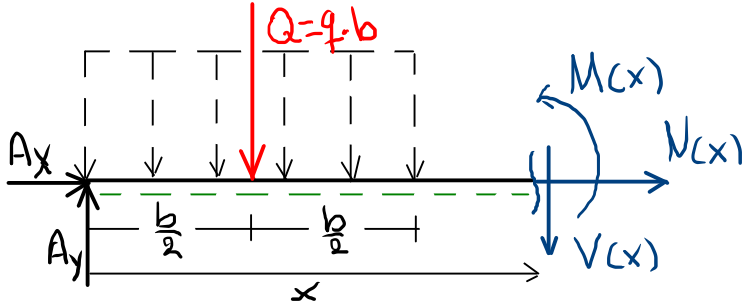


Τομές:

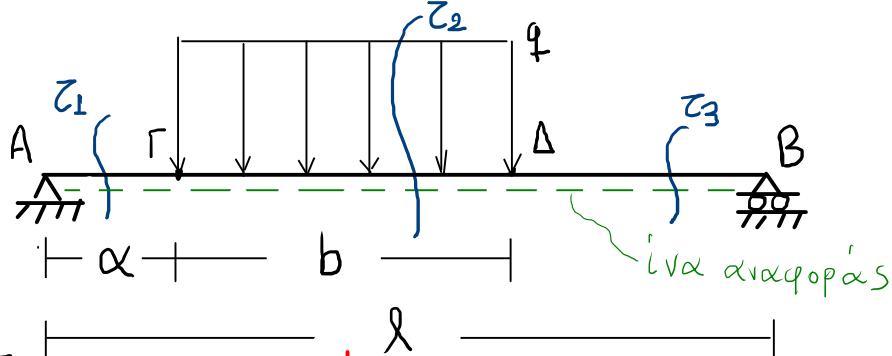
z_1 : τμήμα AC, $0 \leq x < b$



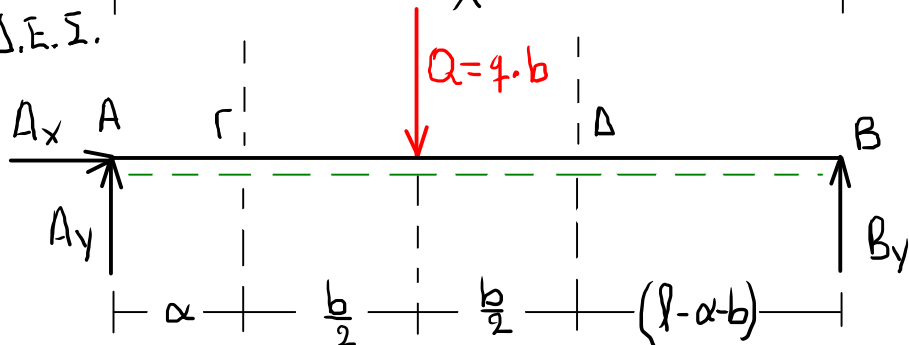
z_2 : τμήμα CB, $b \leq x < l$



Εύρεση A_x, A_y, B_y από ως 3 εξ. ισορ.



Δ.Ε.Σ.

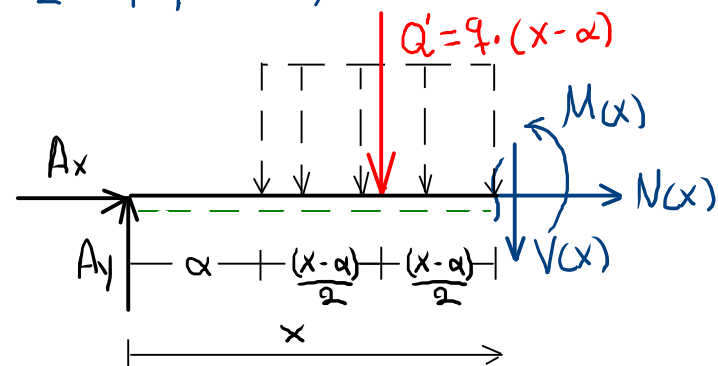


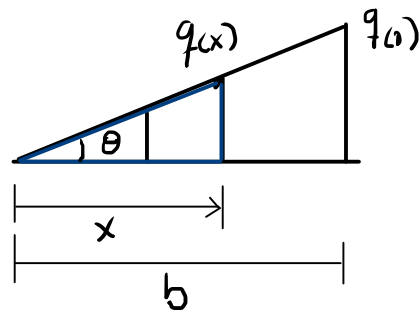
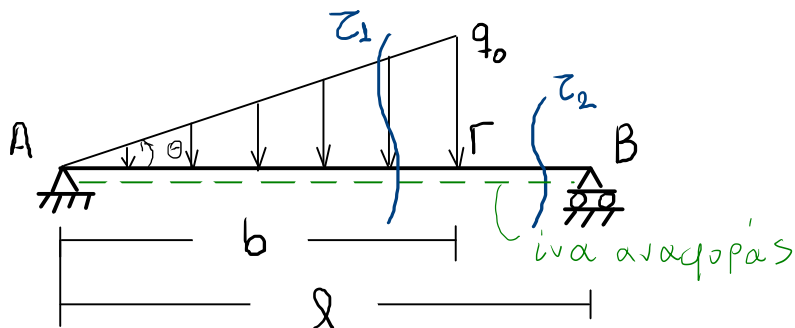
Τομές:

τ_2 : τμήμα ΓΔ, $\alpha \leq x < (\alpha+b)$

τ_1 : ΑΓ, $0 < x < \alpha$
 τ_3 : ΔΒ, $(\alpha+b) \leq x < l$

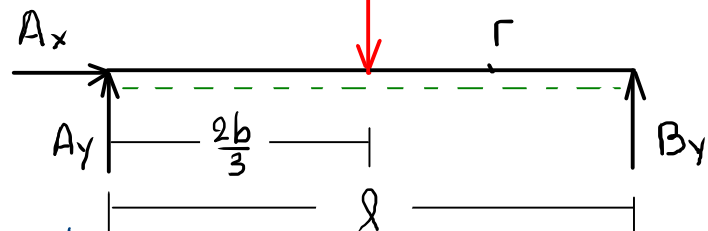
} λύνουμε κατά τα γνωστά (απλές περιπτώσεις)





Δ.Ε.Σ.

$$Q = \frac{1}{2} b q_0$$



$$\frac{q(x)}{q_0} = \frac{x}{b} \Rightarrow q(x) = q_0 \frac{x}{b}$$

για $x=0$: $q(0) = 0$

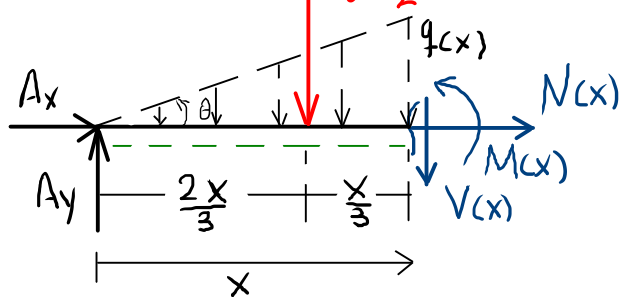
για $x=b$: $q(b) = q_0 \frac{b}{b} = q_0$

Τομές:

z_1 : τμήμα ΑΓ, $0 < x < b$

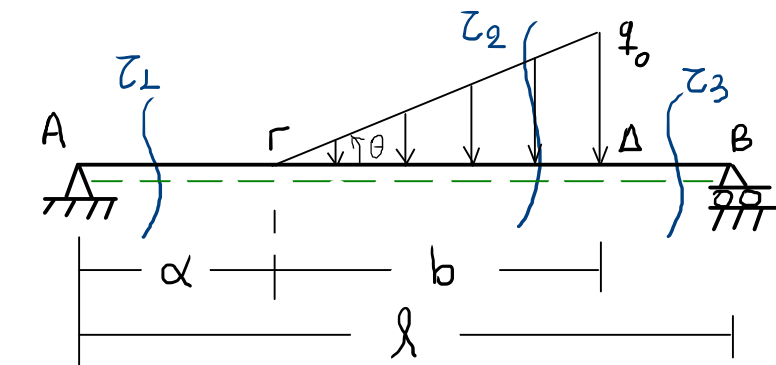
$$Q' = \frac{1}{2} x q(x) = \frac{1}{2} \frac{q_0}{b} x^2$$

$$\tan \theta = \frac{q_0}{b} = \frac{q(x)}{x} \Rightarrow q(x) = \frac{q_0}{b} \cdot x$$

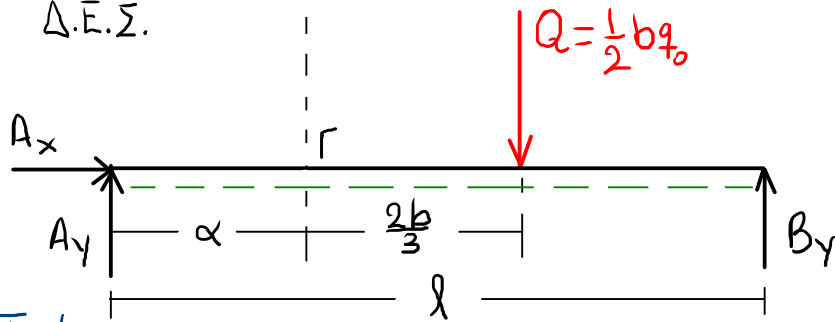


z_2 : ΓΒ, $b \leq x < l$

για ευκολία θα μπορούσαμε να πάρουμε το Δ.Ε.Σ. του κομματιού δεξιά της τομής z_2

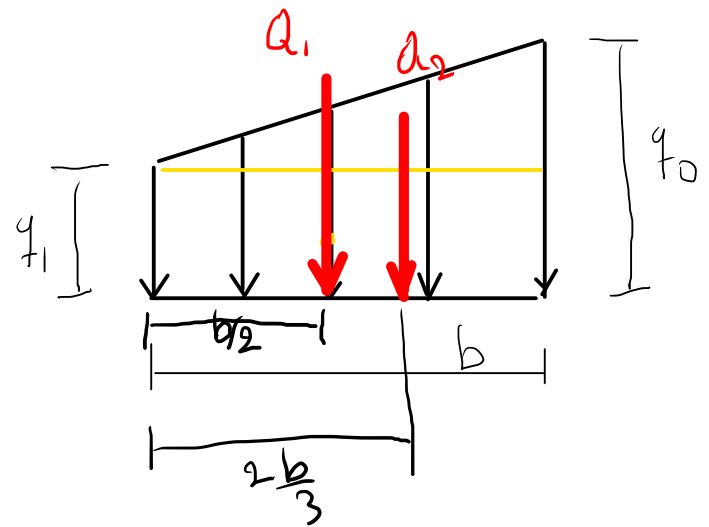
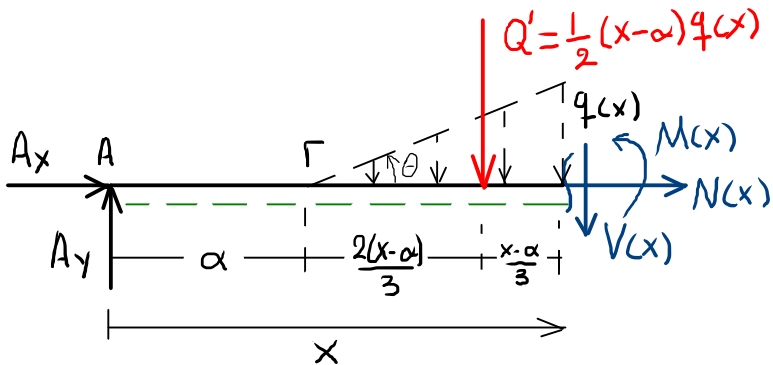


Δ.Ε.Σ.



Τομές

z_2 : ορίζεται $\Gamma\Delta$, $\alpha \leq x < (\alpha+b)$



$$Q_1 = b \cdot q_1$$

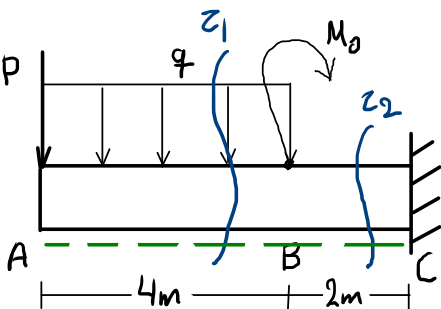
$$Q_2 = \frac{1}{2} b (q_0 - q_1)$$

$$\tan \theta = \frac{q_0}{b} = \frac{q(x)}{x-\alpha} \Rightarrow q(x) = \frac{q_0}{b} (x-\alpha)$$

$z_1: A\Gamma$, $0 < x < \alpha$ } δύναμη και ζεύγος
 $z_3: \Delta B$, $(\alpha+b) \leq x < l$ } και γωνία

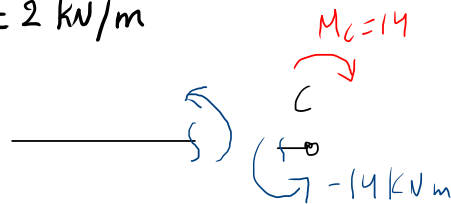
Παράδειγμα

Πρόβλημα με διάφορες φορτίσεις:



Δεδομένα: $P = 4 \text{ kN}$, $M_0 = 42 \text{ kN}\cdot\text{m}$, $q = 2 \text{ kN/m}$

$$Q = q \cdot 4 = 2 \cdot \frac{\text{kN}}{\text{m}} \cdot 4 \text{ m} = 8 \text{ kN}$$



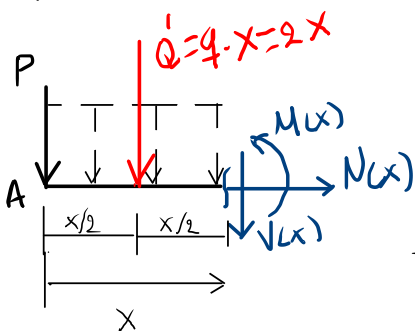
$$\sum F_x = 0 \Rightarrow \underline{C_x = 0}$$

$$\begin{aligned} \sum F_y = 0 &\Rightarrow -P - Q + C_y = 0 \Rightarrow C_y = P + Q \\ &\Rightarrow C_y = 4 + 8 \Rightarrow \underline{C_y = 12 \text{ kN}} \end{aligned}$$

$$\begin{aligned} \sum M_C = 0 &\Rightarrow P \cdot 6 + Q \cdot 4 - M_0 - M_C = 0 \\ &\Rightarrow M_C = P \cdot 6 + Q \cdot 4 - M_0 = 4 \cdot 6 + 8 \cdot 4 - 42 \\ &\Rightarrow M_C = 24 + 32 - 42 \Rightarrow \underline{M_C = 14 \text{ kN}\cdot\text{m}} \end{aligned}$$

Τομές:

$z_1: 0 < x < 4 : (AB)$



$$\sum F_x = 0 \Rightarrow N(x) = 0$$

$$\sum F_y = 0 \Rightarrow -P - Q' - V(x) = 0 \Rightarrow V(x) = -P - Q' = -4 - 2x$$

Boundary conditions for shear force $V(x)$:

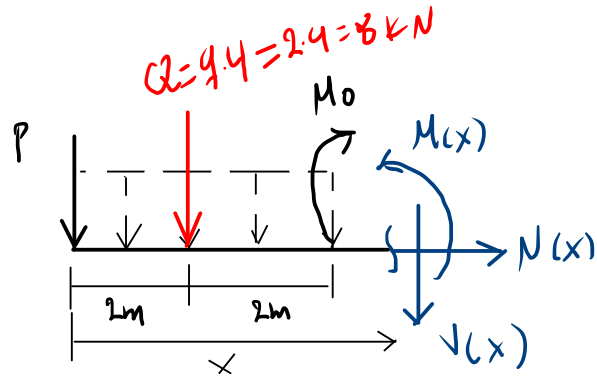
- At A: $x=0, V(0) = -4$
- At B: $x=4, V(4) = -12$

$$\begin{aligned} \sum M_x = 0 &\Rightarrow P \cdot x + Q' \cdot \frac{x}{2} + M(x) = 0 \Rightarrow 4 \cdot x + 2x \cdot \frac{x}{2} + M(x) = 0 \\ &\Rightarrow M(x) = -x^2 - 4x \end{aligned}$$

Boundary conditions for moment $M(x)$:

- At A: $x=0, M(0) = 0 \text{ kN}\cdot\text{m}$
- At B: $x=4, M(4) = -32 \text{ kN}\cdot\text{m}$

$z_2: 4 \leq x < 6$ (BC)



$$(+ \rightarrow) \sum F_x = 0 \Rightarrow N(x) = 0$$

$$(+ \uparrow) \sum F_y = 0 \Rightarrow -P - Q - V(x) = 0 \Rightarrow -4 - 2 \cdot 4 - V(x) = 0$$

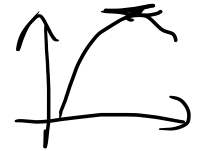
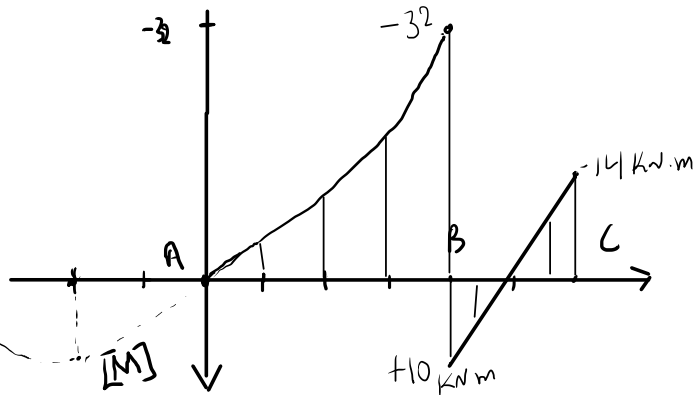
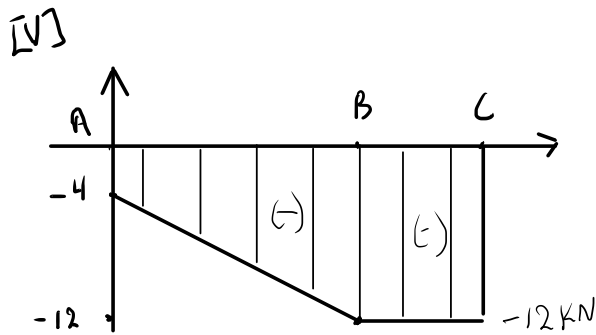
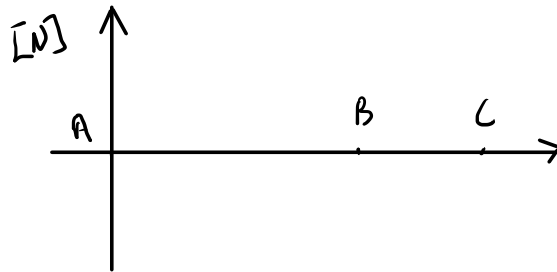
$$\Rightarrow V(x) = -12 \text{ kN}$$

$$+ \curvearrowright) \sum M_x = 0 \Rightarrow P \cdot x + Q(x-2) - M_0 + M(x) = 0$$

$$\Rightarrow 4 \cdot x + 8(x-2) - 42 + M(x) = 0$$

$$\Rightarrow M(x) = 42 - 12x + 16$$

$$\Rightarrow M(x) = 58 - 12x \quad \left| \begin{array}{l} B: x=4, M(4) = 10 \text{ kNm} \\ C: x=6, M(6) = -14 \text{ kNm} \end{array} \right.$$



$$M(x) = -x^2 - 4x$$

$$\frac{dM(x)}{dx} = -2x - 4 = 0$$

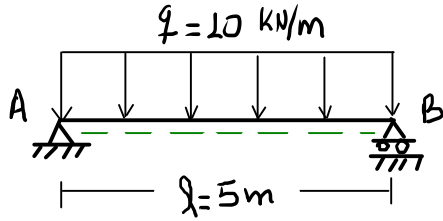
$$\Rightarrow 2x = -4 \Rightarrow x = -2$$

$$\frac{d^2M(x)}{dx^2} = -2 < 0$$

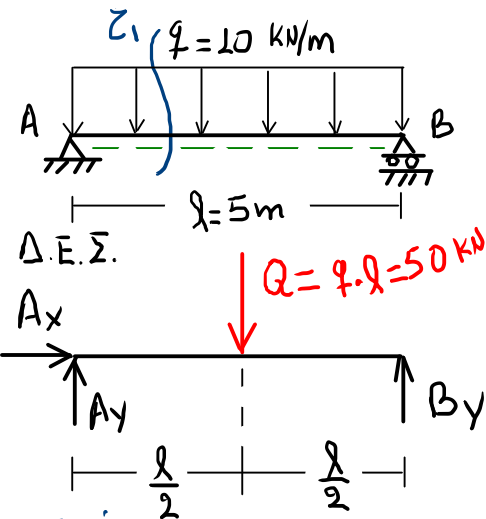
Ex 31 d k p 2.20

Άσκηση:

Να σχεδιάσετε τα διαγράμματα των ενταξικών μεγεθών $[N]$, $[V]$, $[M]$ για τη δοκό του παρακάτω σχήματος.



Λύση



$$\begin{aligned} (+) \sum F_x = 0 &\Rightarrow \underline{A_x = 0} \quad (1) \end{aligned}$$

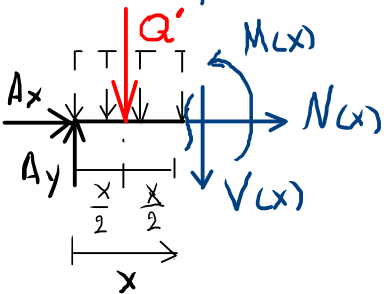
$$\begin{aligned} (+) \uparrow \sum F_y = 0 &\Rightarrow A_y + B_y - Q = 0 \Rightarrow A_y + B_y = 50 \quad (2) \end{aligned}$$

$$\begin{aligned} \curvearrowright \sum M_A = 0 &\Rightarrow -Q \cdot \frac{l}{2} + B_y \cdot l = 0 \Rightarrow B_y = \frac{Q}{2} \Rightarrow \underline{B_y = 25 \text{ kN}} \quad (3) \end{aligned}$$

$$(2) \xrightarrow{(3)} \underline{A_y = 25 \text{ kN}}$$

Τόπος:

$\tau_1: AB, 0 \leq x \leq l$



$$Q' = q \cdot x = 10 \cdot x$$

$$\begin{aligned} (+) \sum F_x = 0 &\Rightarrow A_x + N(x) = 0 \Rightarrow N(x) = -A_x \Rightarrow \underline{N(x) = 0} \end{aligned}$$

$$\begin{aligned} (+) \uparrow \sum F_y = 0 &\Rightarrow A_y - Q' - V(x) = 0 \Rightarrow V(x) = A_y - Q' \Rightarrow \end{aligned}$$

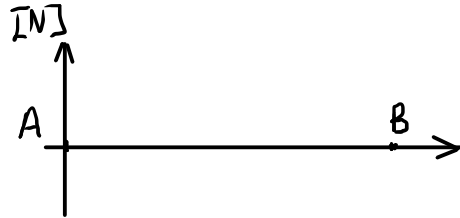
$$\Rightarrow \underline{V(x) = 25 - 10x} \quad \begin{array}{l} \text{A: } x=0 \Rightarrow V(0) = 25 \text{ kN} \\ \text{B: } x=5 \Rightarrow V(5) = -25 \text{ kN} \end{array}$$

$$\begin{aligned} \curvearrowright \sum M_x = 0 &\Rightarrow -A_y \cdot x + Q' \cdot \frac{x}{2} + M(x) = 0 \Rightarrow M(x) = A_y \cdot x - q \cdot \frac{x^2}{2} \end{aligned}$$

$$\Rightarrow \underline{M(x) = 25x - 5x^2} \quad \begin{array}{l} \text{A: } x=0 \Rightarrow M(0) = 0 \\ \text{C: } x=2.5 \Rightarrow M(2.5) = 31.25 \text{ kNm} \\ \text{B: } x=5 \Rightarrow M(5) = 0 \end{array}$$

Διαγράμματα Εντατικών Μεγεθών [N], [V], [M]

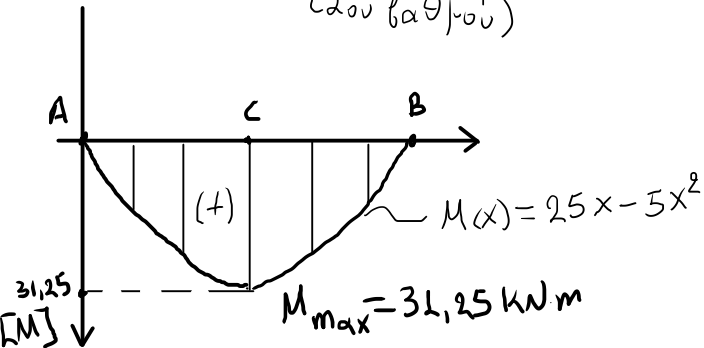
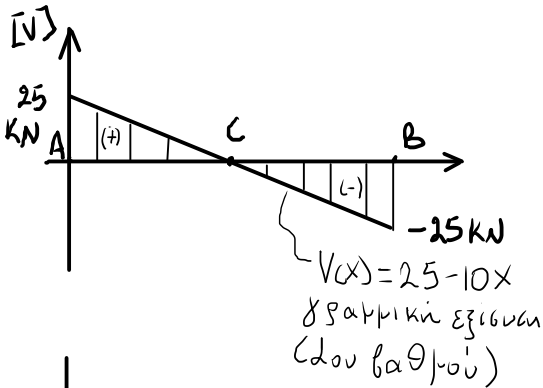
$$\underline{N(x)=0}, \quad \underline{V(x)=25-10x} \quad \left| \begin{array}{l} A: x=0 \Rightarrow V(0)=25 \text{ kN} \\ B: x=5 \Rightarrow V(5)=-25 \text{ kN} \end{array} \right. \quad \underline{M(x)=25x-5x^2} \quad \left| \begin{array}{l} A: x=0 \Rightarrow M(0)=0 \\ C: x=2.5 \Rightarrow M(2.5)=31.25 \\ B: x=5 \Rightarrow M(5)=0 \end{array} \right.$$



$$\left\{ \begin{array}{l} \frac{dM(x)}{dx} = 0 \Rightarrow 25 - 10x = 0 \Rightarrow x = 2.5 \\ \frac{d^2M(x)}{dx^2} = -10 < 0 \text{ ερμειεί τα κοίλα κάτω} \end{array} \right.$$

αναγινώσκω ακροαζου

Επομένως στο $x=2.5 \text{ m}$ η ποσότητα γίνεται μέγιστη $M_{\max} = M(2.5) = 31.25 \text{ kN}\cdot\text{m}$



εξίσωση παραβολής
(2ου βαθμού)